Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

Monday, 9:00–10:30 am, to attend via Zoom email David Jekel (daj@math.ku.dk) Remote

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Jun 8 **Charles-Philippe Diez**, Technion

Obata's rigidity theorem in free probability

In this talk, we will discuss a rigidity phenomenon for Voiculescu's free Poincare inequality under a non-commutative curvature-dimension condition. In the classical setting, on a n-dimensional Riemannian manifold, Lichnerowicz's estimate gives a sharp lower bound $\lambda_1 \geq n$ for the first non-zero eigenvalue of the Laplace-Beltrami operator under a uniform positive Ricci curvature bound $\mathrm{Ric}_g \geq (n-1)g$, and Obata's theorem identifies the round sphere as the unique equality case. Cheng and Zhou later established an infinite-dimensional analogue: on a weighted Riemannian manifold satisfying $CD(K,\infty)$ with $K > 0$, equality in the spectral gap occurs only when a one-dimensional Gaussian space splits off. This result was subsequently extended to the non-smooth RCD framework by Gigli , Ketterer, Kuwada and Ohta. We will explain how an analogous mechanism appears in free probability, in the "Ricci-flat" setting of free difference quotients. We first establish a free Brascamp-Lieb (Hessian-Poincare) inequality, showing that under a suitable curvature condition the free Poincare constant is bounded by the inverse of the convexity parameter, in full analogy with the classical log-concave case. Under the same condition, any extremizer (achieving equality) of the free Poincare inequality must be an affine function of the generators, as in the Ornstein-Uhlenbeck situation. As a consequence, we show that the underlying tracial von Neumann algebra necessarily splits off an abelian, freely complemented semicircular component, which, by Popa's results, is maximal amenable. In higher rank, under additional assumptions, the extremal directions form a free semicircular family, yielding a free product with a free group factor. This provides a free analogue of the classical rigidity phenomenon and highlights new connections between commutation relations, semigroup techniques, non-commutative Dirichlet forms, and free Bakry-Emery theory.