

Probabilistic Operator Algebra Seminar

Organizer: Dan Virgil Voiculescu

Monday, 9:00–10:30 am, to attend via Zoom email David Jekel (djekel@uottawa.ca) remot

Nov 9

Luis Daniel Abreu, University of Vienna

A computable wandering and tracelike vector for modular orbits in the Bergman space

We construct a function Φ such that the orbit under the representation of $\Gamma = PSL(2, Z)$ is an orthonormal basis for the Bergman space with weight $q = 12$. Moreover, we show that Φ is effectively computable as a holomorphic function on the upper half-plane (in the precise sense of computable analysis), by providing an effective procedure. This constructs a wandering and tracelike vector for $PSL(2, Z)$, whose abstract existence was proved by Sir Vaughan Jones in his last paper, where the corresponding construction was left as a problem. The construction of the function, which can also be represented as an orthonormal wavelet in the half-line, reproduces the method used in the signal approximation practice: the off-diagonal decay of the Gramian operator allows to control its restriction to nested finite subsets of $PSL(2, Z)$, where it becomes a finite matrix and can be inverted, leading to a convergent sum where all objects and bounds are computable in the cyclotomic field generated by the 12th-root of unity over the rationals. Since modular actions on the Bergman kernel enter the construction, Φ displays Bergman and modular reminiscences, despite appearing to be modular itself. As suggested by Jones, it provides a computable implementing vector for the abstract anti-isomorphism between the von Neumann algebra $M_{12}(\Gamma)$ and its commutant. With minor adaptations, the construction works for any cocompact Fuchsian group, but the more involved proof of computability is only executed for $PSL(2, Z)$.