

Probabilistic Operator Algebra seminar

Organizer: Dan Virgil Voiculescu

Monday, 9:00–10:30 am, to attend via Zoom email David Jekel (djekel@uottawa.ca) remotely

Oct 26 **Raphael Ponge**, University of Ottawa

Traces on Weak Lorentz Ideals and Nonclassical Weyl Laws

In Connes' noncommutative geometry, traces on operator ideals play the role of the integral. The basic examples are Dixmier traces and, more generally, positive normalized traces on the weak trace class $L_{1,\infty}$. This setting is tailored to classical Weyl laws, where eigenvalues decay like $j^{-1/p}$. Many natural operators, however, obey nonclassical Weyl laws with logarithmic corrections, $\lambda_j \sim c j^{-1/p} (\log j)^q$. Examples include Dirichlet Laplacians on infinite-volume domains, Dirac operators on open manifolds with cusps and assuming the Riemann Hypothesis, the Dirac operator of Connes–Moscovici whose eigenvalues are the imaginary parts of the zeros of the Riemann zeta function. Such operators lie outside $L_{1,\infty}$, in weak Lorentz ideals associated with regularly varying functions. This talk is about traces on these ideals. I will first show that an asymptotic additivity property for eigenvalue partial sums allows us to define Dixmier traces directly in terms of eigenvalue sequences. This leads to a spectral characterization of operators on which all Dixmier traces agree, answering a question of Connes in this setting. I will then go beyond Dixmier traces and consider the whole class of positive normalized traces. The main tool is an extension of Pietsch's correspondence which relates traces on weak Lorentz ideals to invariant functionals on sequence spaces and places Dixmier traces within a broader picture. It yields a spectral characterization of operators on which all positive normalized traces agree. This extends results of Pietsch and Semenov–Sukochev–Usachev–Zanin for $L_{1,\infty}$. Combined with an extension of the Birman–Solomyak perturbation theory, it shows that operators satisfying Weyl laws take the same value on all positive normalized traces. I will also discuss a further notion, hypermeasurability. Finally, I will illustrate these results with examples, including the Riemann Hypothesis, the logarithm of the Laplacian, and products of Laplacians. This is joint work with Yongqiang Tian (Central South University, Changsha, China).